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Cosmic inflation in some viable modified models of f(R) gravity of polynomial- exponential form

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Abstract. In this paper, we present the cosmic inflation scenario in some viable models of the f(R) modified gravity of polynomial- exponential form. Results show that the magnitude of the parameter β in these viable models is at the order of 5.67×10^{-76}, and the time of inflation is at the order of 4.6×10^{-35} s.

1. Introduction

Alan Guth [1] proposed cosmic inflation in 1980 to explain the observable circumstances in the universe. The explosively quick expansion of space-time that occurred a fraction of a second after the Big Bang is referred to as inflation. In a fraction of the second more, inflation slowed to a more steady rate, which it has continued and is again accelerating.The inflationary phase lasted between 10^{-33} and 10^{-32} seconds following the conjectured Big Bang singularity. The cosmos continues to grow after the inflationary phase but at a slower speed. The origin of the cosmos' large-scale structure is explained by inflation theory. The seeds for the growth of structure in the universe are quantum fluctuations in the microscopic inflationary zone [2, 5], which enlarged to a cosmic scale. Cosmological inflation [6-9] explains why the universe appears to be the same in all directions (isotropic), why the cosmic microwave background radiation is dispersed uniformly, why the cosmos is flat, and why no magnetic monopoles have been found. In 2018, Hawking and Thomas Hertog projected that the universe created by eternal inflation on the past boundary would be small and significantly simpler than the endless fractal structure promised by the old eternal inflation hypothesis. This theory [10, 11] does not fully eliminate inflation, but it does aid in the addition of restrictions to the gravity model in order to determine the precise inflation parameters and times following the Big Bang. Einstein's general theory of relativity is generalized with the f(R) modified theory of gravity. The function f(R) is a function of Ricci scalar R. Corrections emerging from a quantum theory of gravity may have inspired some functional forms. Hans Adolph Buchdahl [12] postulated f(R) gravity in 1970 (but φ was used instead of f for the name of the arbitrary function). Following Starobinsky's work on cosmic inflation [13], it

has become a thriving subject of study. This theory can produce a wide range of phenomena by taking various functions; nevertheless, several functional forms can currently be excluded on observational or pathological theoretical grounds. Following the finding of the universe's accelerating expansion, f(R) gravity has been recognized as a possible explanation for the universe's accelerated expansion and structure building without the addition of unknown types of dark energy or dark matter. The Einstein – Hilbert equation is obtained from the Einstein – Hilbert lagrangian in the General Theory of Relativity (GTR) as follows,

$$S_{E-H} = \frac{1}{2k^2} \int d^4x \sqrt{-g}\left(R - 2\Lambda\right) + S_M$$

(1)

Here R is Ricci scalar.

In modified theories of gravitation, the Einstein – Hilbert lagrangian becomes,

$$S_J\left(g_{ab}\right) = \frac{1}{2k^2} \int f\left(R\right) d^4x \sqrt{-g} + S_M$$

(2)

Here f(R) is a nonlinear function of scalar Ricci R. S_M is an action of material field.

In 2015, we first introduced a modified model of gravitation of polynomial-exponential form to explain the universe's accelerating expansion [14]. In this model, f(R) is a function of R in polynomial-exponential form with two unknown parameters are alpha and beta, determined from observed cosmological data. The alpha parameter is constrained by data in the Solar system [15], while the beta is determined only by data from cosmological inflation. Therefore, in the paper, we investigate cosmological inflation in some viable models of modified gravitation of polynomial- exponential form to constrain the beta parameter. This paper is structured as follows. In Section 2 we present the basics of slow-roll inflation; in section 3, we report the cosmic inflation in some viable models of f(R) modified gravity of polynomial-exponential form; section 4 is this paper's conclusion.

2. Basic on slow – roll inflation

Cosmic inflation is an epoch in which the universe expands at a faster rate than usual. In the most basic models, inflation is driven by a single canonical scalar field called the inflaton and a smooth potential that rolls slowly. In the inflationary period, the quantum fluctuations of the scalar field were extended to the cosmological scales, which caused the inhomogeneity and anisotropy observed in the large-scale structure (LSS) and background radiation (CMB) of the universe [16-17]. In the slow-rolling inflation scenario, the inflation is generated by a scalar field rolling down a hill of potential energy instead of tunneling out of a false vacuum. When the rate of rolling down the energy potential hill of the scalar field is very slow compared to the expansion rate of the universe, inflation begins to occur.[18]. As the energy potential hill becomes steeper, cosmic inflation stops and reheating can occur. To better understand slow-roll inflation, we'll examine the Klein – Gordon equation, often known as the motion equation of the canonical homogeneous inflaton field ϕ,

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

(3)

Where H represents the Hubble parameter, ϕ represents the scalar potential, and the dot represents the derivative with respect to cosmic time. In the inflation period, when the universe is dominated by the inflaton field, we also employ the flat Friedman equation,

$$3H^2 m_p^2 = \frac{1}{2}\dot{\phi}^2 + V \tag{4}$$

here $m_p = \sqrt{\dfrac{\hbar c}{8\pi G}}$ is the reduced Planck mass.

We also use the slow-roll parameter $\varepsilon \equiv -\dot{H}/H^2$, in inflation period, it should be smaller than unity.

In f(R) modified gravity, modified Friedmann becomes [14],

$$3FH^2 = \frac{FR - f}{2} - 3H\dot{F} \tag{5}$$

with

$$F = \frac{\partial}{\partial R} f(R), \quad \dot{F} = \frac{\partial}{\partial t} F \tag{6}$$

In addition, inflation has to endure for a certain amount of time, such that CMB modes measured must expand by at least 50 – 70 times. Thus the number of e-foldings in a given mode should be defined as the number of e-folds that the i-mode has grown from the horizon crossing until the end of inflation,

$$N(\phi_i) = \ln \frac{a_f}{a(\phi_i)} = \int_{t_i}^{t_f} H dt \quad \text{with} \quad i = aH \tag{7}$$

here t is the time that the i-mode crosses the horizon. We may express the slow-roll parameters using the e-folding number as,

$$dN = Hdt = d\ln a, \quad \varepsilon = -\frac{d\ln H}{dN}, \quad \varepsilon_{i+1} = \frac{d\ln \varepsilon_i}{dN} \tag{8}$$

3. Cosmic inflation in some viable models of f(R) modified gravity of polynomial-exponential form

In the section, we study the cosmological inflation in some viable models of f(R) modified gravity of polynomial exponential form to value the parameter of β and inflation time. The lagrangian of the model as follows [14],

$$f(R) = R + a + \frac{\alpha}{R^m}\left(1 + bR^2 + cR^3\right)e^{-\beta R^n} \tag{9}$$

Here α, β are positive constants, m, n, a, b, c are constants.

In case $\alpha = 0$ or $R \to \infty$ it is Einstein theory. In this article, we only limit our investigation to the following cases,

Case 1:

$a = 2\Lambda, b = c = 1, m = n = 1$, and α, β are positive constants less than unit.

At this equation (9) becomes,

$$f(R) = R - 2\Lambda + \frac{\alpha}{R}\left(1 + R^2 + R^3\right)e^{-\beta R}$$

(10)

Euler expansions get approximate: $e^{-\beta R} = 1 - \beta R$

$$f(R) = R - 2\Lambda - \alpha\beta(R^3 + R^2 + 1) + \alpha(R^2 + R + \frac{1}{R})$$

(11)

Inflation period R is very large,

$$f(R) = R + \alpha R^2 - \alpha\beta R^3$$

(12)

From modified Friedmann's equation in f(R) gravity,

$$3FH^2 = \frac{FR - f}{2} - 3H\dot{F} \qquad , \qquad F = \frac{\partial}{\partial R}f(R) \quad , \quad \dot{F} = \frac{\partial}{\partial t}F$$

The slow – roll parameter becomes,
,

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{-48\beta H^2 + 36\beta + 2}{192\beta H^2 + 18\beta - 3}$$

(13)

The Hubble parameter satisfies,

$$H^2 \gtrsim \frac{18\beta + 5}{240\beta}$$

(14)

At time t=t_f , slow – roll parameter become homogeneous and inflation stopped,

$$-\frac{\dot{H}_f}{H^2} \simeq 1 \Rightarrow \dot{H}_f \simeq -\sqrt{\frac{18\beta + 5}{240\beta}}$$

(15)

$$t_f = \frac{H_f}{\sqrt{\dfrac{18\beta + 5}{240\beta}}} + t_i$$

(16)

E-foldings index,

$$N \simeq H_f \left(t_f - t_i \right) - \frac{1}{2}\left(\sqrt{\frac{18\beta+5}{240\beta}} \right)\left(t_f - t_i \right)^2 \tag{17}$$

$$N \simeq H_f \frac{H_f}{\sqrt{\frac{18\beta+5}{240\beta}}} - \frac{1}{2}\left(\sqrt{\frac{18\beta+5}{240\beta}} \right)\left(\frac{H_f}{\sqrt{\frac{18\beta+5}{240\beta}}} \right)^2$$

$$\simeq \frac{1}{2}\frac{H_f{}^2}{\sqrt{\frac{18\beta+5}{240\beta}}} \simeq \frac{1}{2\varepsilon\left(t_f \right)} \tag{18}$$

To claim a successful inflation model, N needs to satisfy 70> N > 50, we determine the value of the parameter β,

$$70 > \frac{1}{2\varepsilon\left(t_f \right)} > 50 \tag{19}$$

$$140 > \frac{1}{\varepsilon\left(t_f \right)} > 100$$

$$\frac{203}{4992H^2 - 3582} < \beta < \frac{283}{6528H^2 - 5022} \tag{20}$$

In time of inflation,

$$H = \sqrt{\tfrac{8}{3}\pi G\rho} = 1.1\times10^{37} \text{ s}^{-1}$$

We have,

$$3,36.10^{-76} < \beta < 3,58.10^{-76} \tag{21}$$

Case 2:

$a = 2\Lambda, b = c = 1, m = 2, n = 1$, and α, β are positive constants.

At this equation (9) becomes,

$$f\left(R \right) = R - 2\Lambda + \frac{\alpha}{R^2}\left(1 + R^2 + R^3 \right)e^{-\beta R} \tag{22}$$

Same as the calculation above. We have,

$$6,85.10^{-76} < \beta < 6,97.10^{-76} \tag{23}$$

Case 3:

The model has the same form as in the above cases,

$$f(R) = R - 2\Lambda + \frac{\alpha}{R^2}\left(1 + R^2 + R^3\right)e^{-\beta\left(R+R^2\right)}$$
(24)

We have, $$6,56.10^{-76} < \beta < 6,68.10^{-76}$$
(25)

With the three models we just calculated, the value of the parameter in each model is quite small and in a narrow range. To better understand the cosmic inflation in these models, we consider the slow–rolling parameter and the inflation time.

Mean value of β in all three models,

$$\beta = 5,67.10^{-76}$$
(26)

We have a low- rolling parameter corresponding to,

$$\varepsilon = 0,0046$$
(27)

Inflation time is calculated based on the slow-rolling parameter,

$$\varepsilon \simeq \frac{\dot{H}}{H^2}$$
(28)

We have,

$$\frac{\frac{dH}{dt}}{H^2} = -\varepsilon \Rightarrow \frac{dH}{H^2} = -\varepsilon dt \Rightarrow \frac{1}{H} = \varepsilon t$$
(29)

$$t = \frac{1}{\varepsilon H} \simeq 4,6.10^{-35}\,s$$
(30)

The timing of inflation in these model classes is quite close to the standard inflation models proposed earlier, which is quite similar to the CMB and gravitational wave measurements are about 10^{-35} s.

4. Conclusion

In conclusion, we present three modified models of f(R) gravities in a polynomial–exponential form that satisfies the possible conditions of a standard inflation model to combine with the Friedmann equation to calculate the value of the parameter beta and the lasted time of cosmic inflation. Results obtained beta parameter is about $5,67.10^{-76}$, and the lasted time of cosmic inflation is in order of $4,6.10^{-35}\,s$. The value of beta is less than the alpha one order. The lasted time of cosmic inflation is the same order as that in current cosmic models.

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